

2.4) $f(x)$ defined $\forall x \in (a-\delta, a+\delta)$ - except perhaps $x=a$.
 for every (in) (element of) $x=a$.
 i.e. $f(a)$ doesn't need to exist

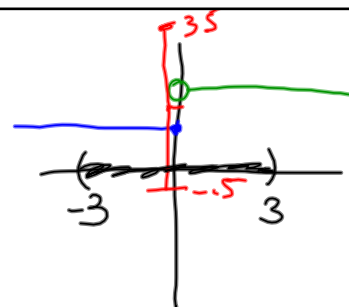
$\lim_{x \rightarrow a} f(x) = L$ iff

(1) $\forall \epsilon > 0$,
 epsilon

(2) $\exists \delta > 0$
 there exists delta

(3) such that $\forall x \in (a-\delta, a+\delta) \Rightarrow f(x) \in (L-\epsilon, L+\epsilon)$
 (except $x=a$)

$$f(x) = \begin{cases} 1 & \text{if } x \leq 0 \\ 2 & \text{if } x > 0 \end{cases}$$



→ choose $\epsilon = 2$

is there a δ so that

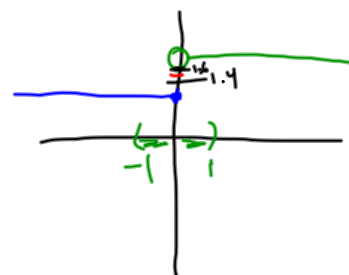
every x in $(0-\delta, 0+\delta)$

has an "f" value in $(-1.5, 3.5)$?

claim: pick $\delta = 3$ ✓

hypothesis:
 $\lim_{x \rightarrow 0} f(x) = 1.5$

$$f(x) = \begin{cases} 1 & \text{if } x \leq 0 \\ 2 & \text{if } x > 0 \end{cases}$$



→ choose $\epsilon = .1$

select $\delta = 1$

$f(\text{every } x \text{ in the left part}) = 1$ oops is NOT in $(1.4, 1.6)$

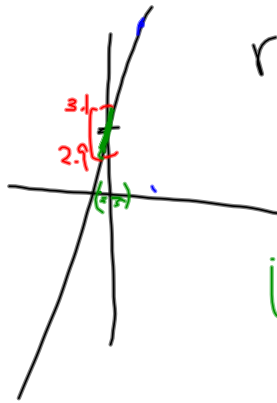
hypothesis:
 $\lim_{x \rightarrow 0} f(x) = 1.5$

$$2.11 \quad f(x) = 2x + 3, \quad \lim_{x \rightarrow 0} f(x) = \underline{\underline{3}}$$

2) suppose $\epsilon = .1$

range of y values $(f(a) - \epsilon, f(a) + \epsilon)$

$$= (2.9, 3.1)$$

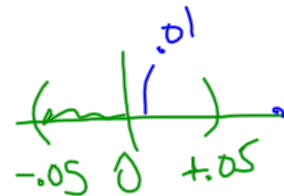


i) can we find c so that $f(c) = 2.9$?

$$2(c) + 3 = 2.9$$

$$2c = -.1$$

$$c = \underline{\underline{-.05}}$$



iii)
 $f(.01) =$
 $2(.01) + 3 =$

$$3.02$$

is this in
 $(2.9, 3.1)$?

ii) can we find a y so $f(y) = 3.1$?

$$2y + 3 = 3.1$$

$$2y = .1$$

$$y = .05$$

suppose
 our x -domain on
 x -interval



2.5/ Continuity

AGAP.... Limits

'new' definitions of

- horizontal asymptotes

e.g. if $\lim_{x \rightarrow \pm\infty} f(x) = L$, then $y = L$ is a h.a.

- vertical asymptotes

e.g. if $\lim_{x \rightarrow a} f(x) = \pm\infty$ they
 $x = a$ is a v.a.

2.5) A function $f(x)$ is continuous at $x=a$
iff

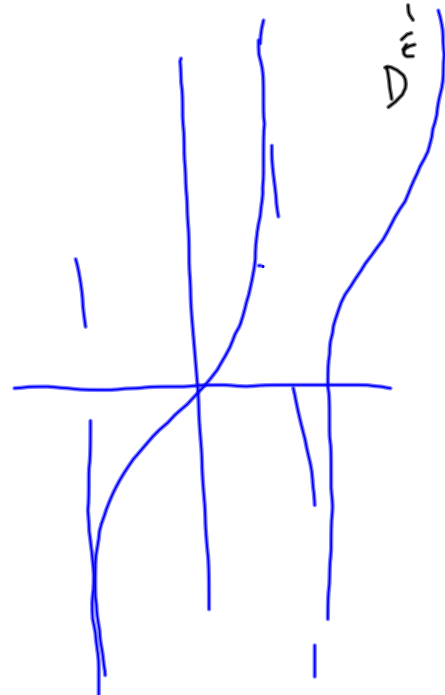
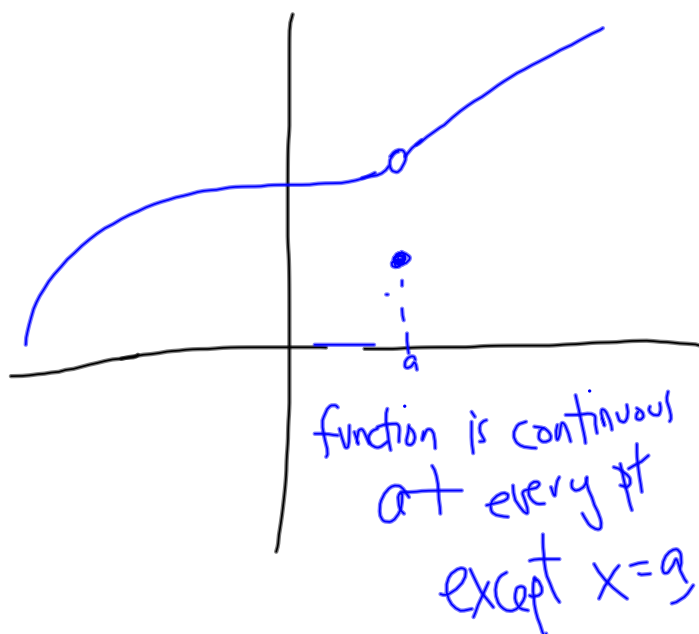
the following 3 conditions are satisfied.

1) $f(a)$ is defined.

2) $\lim_{x \rightarrow a} f(x)$ must exist.

3) $\lim_{x \rightarrow a} f(x) = f(a)$

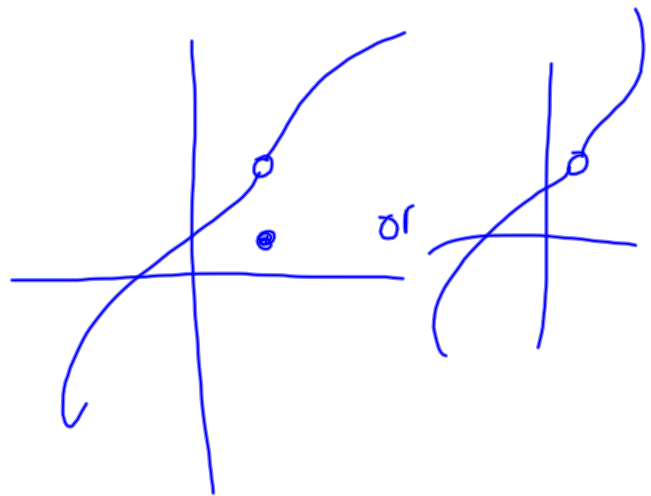
1) $f(a)$ def 2) \lim exists 3) NOT SATISFIED



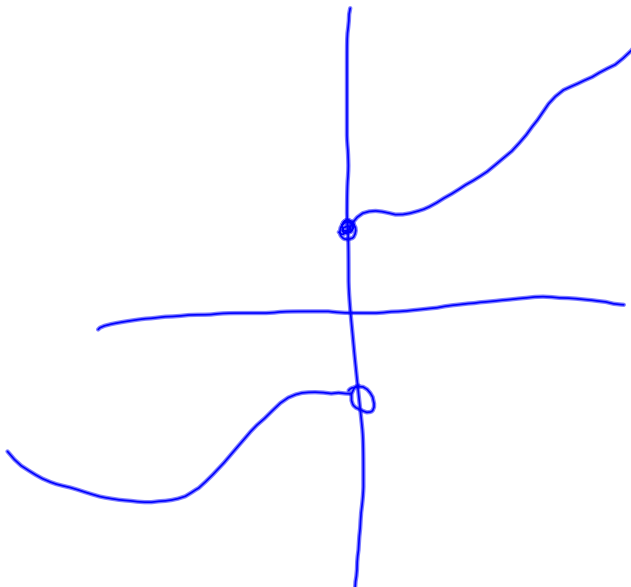
3 types of DISCONTINUITY

1) Removable

→ can make continuous
at a point by
(re)defining function
value



2) jump discontinuity



3) infinite discontinuity

